

(3) June 2007

$$1) \begin{aligned} y &= x^3(x+1)^5 = uv \\ u &= 3x^2 \quad v = 5(x+1)^4 \\ y &= x^3[5(x+1)^4 + (x+1)^5]3x^2 \\ &= (x+1)^4 x^2 (5x+3(x+1)) \\ &= x^2(x+1)^4 (8x+3) \end{aligned}$$

$$2) \quad y = (3x^4 + 1)^{\frac{1}{2}} \quad y' = \frac{1}{2}(3x^4 + 1)^{-\frac{1}{2}} (12x^3) \\ = 6x^3(3x^4 + 1)^{-\frac{1}{2}}$$

$$\begin{aligned} 2) \quad & -2x-1 \quad -4x+3 \quad 2x+1 \\ & A \quad B \quad C \end{aligned}$$

A)  $2x+1 = -4x+3$   
 $6x = 2 \quad x = \frac{1}{3}$   
 B)  $2x+1 = 4x-3$   
 $2x = 4 \quad x = 2$

Here  $x < 2$  or  $x > \frac{1}{3}$   
 or  $\frac{1}{3} < x < 2$

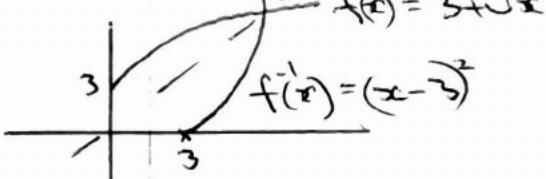
$$3) i) f(x) = 3 + 5x \quad f(16) = 3 + 15 = 16$$

$$f(1) = 3 + 4 = 7$$

$$ii) x \rightarrow 5 \rightarrow +3 = f(x)$$

$$f(x) = (x-3)^2, \quad x > 3$$

$f(x) = 3 + 5x$



Reflection in line  $y=x$

$$4) \int_0^{13} (2x+1)^{\frac{1}{3}} dx = \frac{(2x+1)^{\frac{4}{3}}}{2 \times \frac{4}{3}}$$

$$I(13) = \left. \frac{27}{8} \right|_{0}^{13} = \frac{3}{8} \times 81$$

$$I(0) = \left. \frac{1}{8} \right|_{0}^{13} = \frac{3}{8}$$

$$I = \frac{3}{8} \times 81 - \frac{3}{8} = 30$$

$$4ii) \begin{array}{rccccc} x & 0 & 6.5 & 13 \\ y & 1 & \frac{14}{3} & 3 \\ & & (2.4101) & \end{array}$$

$$\begin{aligned} I &= \frac{1}{3} \times 6.5 (1 + 3 + 4 \times 2.4101) \\ &= 29.6 \text{ to 3sf} \end{aligned}$$

$$5) i) M = 240e^{-0.04t}$$

$$\text{if } t=0 \quad m=240 \text{ initial mass}$$

$$\text{if } m=120 \Rightarrow 120 = 240e^{-0.04t}$$

$$\ln 0.5 = -0.04t$$

$$t = 17.3 \text{ yrs to 3sf}$$

$$ii) \frac{dm}{dt} = -240 \times 0.04 e^{-0.04t} = -2.1 \text{ (given)}$$

$$\text{so } e^{-0.04t} = \frac{-2.1}{-240 \times 0.04} = 0.21875$$

$$t = \frac{\ln 0.21875}{-0.04} = 38.4 \text{ yrs to 3sf}$$

$$6) \int 6e^{2x} + x \, dx = 3e^{2x} + \frac{x^2}{2}$$

$$I(a) = 3e^{2a} + \frac{a^2}{2} \quad I(0) = 3$$

$$\text{so } 3e^{2a} + \frac{a^2}{2} - 3 = 42$$

$$3e^{2a} = 45 - \frac{a^2}{2}$$

$$e^{2a} = 15 - \frac{a^2}{6}$$

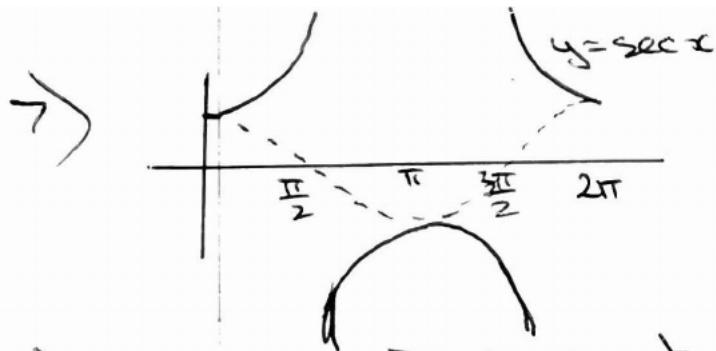
$$2a = \ln(15 - \frac{a^2}{6}) \quad a = \frac{1}{2} \ln(15 - \frac{a^2}{6})$$

$$ii) a_{n+1} = \frac{1}{2} \ln(15 - \frac{a_n^2}{6})$$

$$a_0 = 1 \quad a_1 = 1.348$$

$$a_2 = 1.3438 \quad a_3 = 1.3439$$

$$\text{so } a = 1.344 \text{ to 3dp}$$



$$\text{i)} \sec x = 3 \Rightarrow \cos x = \frac{1}{3}$$

$$x = 70.5^\circ + 289.5^\circ$$

$$= 1.23 + 5.05 \text{ radians}$$

$$\text{iii)} \sec \theta = 5 \cos \theta$$

$$\frac{1}{\cos \theta} = \frac{5}{\sin \theta} \Rightarrow \sin \theta = 5 \cos \theta$$

$$\tan \theta = 5 \quad \theta = 78.7^\circ + 258.7^\circ$$

$$= 1.3734515 \text{ radians}$$

Ans 1.37 + 4.52 radians to 3sf

$$\text{8)} \quad y = \frac{4(\ln x - 3)}{4(\ln x + 3)} = \frac{u}{v} \quad u' = \frac{4}{x} \\ v' = \frac{4}{x}$$

$$\frac{dy}{dx} = \frac{(4(\ln x + 3))\frac{4}{x} - (4(\ln x - 3))\frac{4}{x}}{(4(\ln x + 3))^2}$$

$$= \frac{24}{x(4(\ln x + 3))^2} = \frac{24}{x(4(\ln x + 3))^2}$$

$$\text{i)} \text{ if } y=0 \quad 4\ln x - 3 = 0$$

$$x = e^{\frac{3}{4}}$$

$$\frac{dy}{dx} = \frac{24}{e^{\frac{3}{4}}(4\ln e^{\frac{3}{4}} + 3)^2}$$

$$= \frac{24}{e^{\frac{3}{4}} + 36} = \frac{2}{3e^{\frac{3}{4}}} \text{ or } \frac{2}{3}e^{-\frac{3}{4}}$$

$$\text{iii)} \quad V = \pi \int_1^4 y^2 dx = \pi \int_1^4 \frac{4}{x(4\ln x + 3)^2} dx$$

$$= \frac{\pi}{6} \int_1^4 \frac{24}{x(4\ln x + 3)^2} dx$$

$$= \frac{\pi}{6} \left( \frac{4(\ln x - 3)}{4(\ln x + 3)} \right) \text{ from part i)}$$

$$\text{V(e)} = \frac{\pi}{6} \left( \frac{4(\ln e - 3)}{4(\ln e + 3)} \right) = \frac{\pi}{42}$$

$$\text{V(i)} = -\frac{\pi}{6} \quad V = \frac{\pi}{42} + \frac{\pi}{6} = \frac{8\pi}{42} = \frac{4\pi}{21}$$

$$\text{a)} \text{ i) Use } \tan(A+B) + \tan(A-B)$$

$$\text{LHS} = \left( \frac{\tan \theta + \sqrt{3}}{1 - \sqrt{3} \tan \theta} \right) \left( \frac{\tan \theta - \sqrt{3}}{1 + \sqrt{3} \tan \theta} \right)$$

$$= \frac{\tan^2 \theta - 3}{1 - 3 \tan^2 \theta} = \text{RHS}$$

$$\text{ii) Use RHS of part i) } + 1 + t^2 = \sec^2 \theta$$

$$\frac{\tan^2 \theta - 3}{1 - 3 \tan^2 \theta} = 4(1 + \tan^2 \theta) - 3$$

$$t^2 - 3 = (1 + 4t^2)(1 - 3t^2)$$

$$t^2 - 3 = 1 + t^2 - 12t^4$$

$$12t^4 - 4 = 0 \Rightarrow 3t^4 = 1$$

$$\tan \theta = \pm \frac{1}{\sqrt{3}}$$

$$\tan \theta = \pm \frac{1}{\sqrt{3}} \quad \theta = 37.2^\circ \text{ or } 142.8^\circ$$

$$\text{iii) Using part i)}$$

$$\frac{t^2 - 3}{1 - 3t^2} = k^2$$

$$t^2 - 3 = k^2(1 - 3t^2)$$

$$t^2(1 + 3k^2) - 3 - k^2 = 0$$

$$\text{discriminant} = b^2 - 4ac$$

$$= 0 - 4(1 + 3k^2)(-3 - k^2)$$

$$= (1 + 3k^2)(12 + 4k^2)$$

This expression is  $> 0$  for all values of  $k$  as  $k^2 > 0$  so there are 2 roots

